# A Cauchy Distribution based Video Watermark Detection for H.264/AVC in DCT Domain

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***Abstract*—Compared with Generalized Gaussian distribution (GGD), Cauchy distribution is superior to describe the statistical distribution of the Intra-coded DCT coefficients in H.264/AVC. For the bipolar additive watermark in H.264/AVC video stream, a Cauchy distribution based detection algorithm is proposed by ternary hypothesis testing. Experimental results show that the proposed approach can achieve more than 80% on average for the accuracy of watermark detection.**

## I. INTRODUCTION

Digital watermarking is an important technique for the copyright protection of digital video. Besides watermark embedding, its detection or extraction is also indispensable. Yet, most watermark detection or extraction schemes are designed for some specific embedding algorithms [1]. It is still far away from generic detection and extraction. In image watermarking, a general assumption is often implied about the statistical distribution of those coefficients, which may carry watermarks. The most representative works are generalized Gaussian distribution (GGD) based watermark detection for additive [2] or multiplicative [3] watermark, either in spatial domain or transform domain.

For the latest video standard H.264/AVC, many new coding feature tools such as variable block size motion estimation, multiple intra/inter coding modes and new entropy coding are adopted to improve its compression efficiency. Many distributions such as Gaussian, Laplacian and Cauchy are used to describe the statistical distribution of intra-coded DCT coefficients in H.264/AVC [4]. Nejat .et al proposed a frame bit allocation for the H.264/AVC video coder via Cauchy-density-based rate and distortion models [5]. By exploiting GGD to model these AC coefficients, a likelihood ratio test based theoretical framework is developed for watermark detection [6]. However, it is computation intensive and can not realize watermark extraction. The watermark detection/extraction for H.264/AVC video stream is worthy of further investigation by making full use of the statistical distribution of DCT coefficients.

Motivated by the GGD-based watermark detection in [2], a robust detection scheme of bipolar additive watermarks is proposed for H.264/AVC video stream. The novelties and contributions lie in the nonparametric hypothesis testing of both Cauchy distribution and GGD for H.264/AVC intra-coded DCT coefficients, and further investigation to achieve blind extraction of additive bipolar watermark by ternary hypothesis testing.

## II. THE CAUCHY-DISTRIBUTION MODAL FOR H.264/AVC INTRA-CODED DCT COEFFICIENTS

In this section, maximum likelihood estimation (MLE) is adopted to estimate the parameters of Cauchy distribution for the intra-coded DCT coefficients of H.264/AVC, and $\chi^2$ and K-S test are used for hypothesis testing.

### A. Cauchy distribution model

For Cauchy distribution, its probability density function (PDF) can be written as

$$f(x) = \frac{1}{\pi} \cdot \frac{\lambda}{\lambda^2 + (x-a)^2} \qquad (1)$$

where $\lambda$ and $a$ are shape and position parameter, respectively ($-\infty < a < \infty$, $\lambda > 0$). For a random variable with such a *PDF*, it is denoted as X~ $C(\lambda, a)$. Fig.1 shows a typical distribution of the AC coefficients for an 8×8 block (*Akiyo* sequence). It is symmetric around zero.

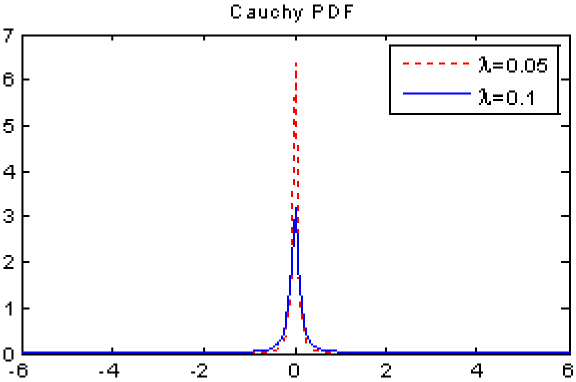

Fig.1. the PDF of Cauchy distribution ( $a = 0$ )

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For the statistics of DCT coefficients, $a$ will be zero [7]. Therefore, only one parameter $\lambda$ is necessary to be estimated. Its final result is derived as follows by MLE [9]:

$$\frac{1}{n}\sum_{i=1}^{n}\frac{2}{1+(x_i/\hat{\lambda})^2}-1=0 \qquad (2)$$

### *B. Non-parametric hypothesis test*

In the following, MLE is used to estimate the parameters of GGD and Cauchy distribution, and then both $\chi^2$ and K-S hypothetic tests [9, 10] are used to verify which is optimal for the model of DCT coefficients of H.264/AVC. Three typical test sequences such as *Akiyo*, *Foreman* and *News* are encoded with JM8.6 reference code. The parameters of H.264/AVC encoder is set as follows: frame rate = 30 f/s, GOP=IIIII, QP=16. For every sequence, hypothesis testing is performed on the DCT coefficients of its first ten frames. The significant level is set with 0.05. Experimental results are summarized in Table I after statistical software SPSS. For most video frames, the K-S and $\chi^2$ values of Cauchy distribution are less than those of GGD. It is obvious that Cauchy distribution is superior to GGD, when they are used to model the intra-coded DCT coefficients of H.264/AVC. However, we might also find several error samples, such as the sixth frame of *Akiyo* sequence. It can be interpreted as singular samples in statistical theory [9].

TABLE I. THE HYPOTHESIS TESTING OF GGD AND CAUCHY DISTRIBUTION OF DCT COEFFICIENTS

| Video | GGD | | Cauchy distribution | | |
|---|---|---|---|---|---|
| (Frame No.) | $\chi^2$ | K-S | $\hat{\lambda}$ | $\chi^2$ | K-S |
| Foreman(1) | 5868.37 | 0.998 | 0.108 | 1760.37 | 0.994 |
| Foreman(2) | 5628.60 | 0.998 | 0.107 | 1717.87 | 0.992 |
| Foreman(3) | 5531.65 | 0.999 | 0.107 | 1706.99 | 0.992 |
| Foreman(4) | 5565.99 | 0.999 | 0.108 | 1887.59 | 0.991 |
| Foreman(5) | 6957.27 | 0.999 | 0.109 | 1775.74 | 0.992 |
| Foreman(6) | 5869.37 | 0.998 | 0.108 | 1735.56 | 0.993 |
| Foreman(7) | 5954.20 | 0.999 | 0.110 | 2022.76 | 0.993 |
| Foreman(8) | 3712.97 | 0.999 | 0.109 | 1846.22 | 0.993 |
| Foreman(9) | 3992.86 | 0.999 | 0.109 | 1653.25 | 0.991 |
| Foreman(10) | 4102.58 | 0.999 | 0.110 | 1969.03 | 0.990 |
| Akiyo(1) | 3768.13 | 0.994 | 0.104 | 344.417 | 0.983 |
| Akiyo(2) | 3610.84 | 0.994 | 0.104 | 348.265 | 0.983 |
| Akiyo(3) | 3449.32 | 0.994 | 0.104 | 329.468 | 0.983 |
| Akiyo(4) | 3943.86 | 0.994 | 0.104 | 341.288 | 0.983 |
| Akiyo(5) | 3963.61 | 0.994 | 0.105 | 351.103 | 0.983 |
| Akiyo(6) | 3821.26 | 0.994 | 0.105 | 387.267 | 0.999 |
| Akiyo(7) | 3930.06 | 0.994 | 0.104 | 365.663 | 0.983 |
| Akiyo(8) | 3876.36 | 0.994 | 0.104 | 381.509 | 0.983 |
| Akiyo(9) | 3708.59 | 0.994 | 0.104 | 362.541 | 0.983 |
| Akiyo(10) | 3656.02 | 0.994 | 0.105 | 359.734 | 0.983 |
| News(1) | 6807.50 | 0.996 | 0.120 | 3573.17 | 0.988 |
| News(2) | 7393.04 | 0.996 | 0.120 | 3547.83 | 0.988 |
| News(3) | 7418.45 | 0.996 | 0.120 | 3552.01 | 0.988 |
| News(4) | 7460.37 | 0.996 | 0.120 | 3365.25 | 0.988 |
| News(5) | 8004.88 | 0.996 | 0.119 | 3399.94 | 0.989 |
| News(6) | 7775.77 | 0.996 | 0.120 | 3346.85 | 0.989 |
| News(7) | 7711.35 | 0.996 | 0.120 | 3458.81 | 0.988 |
| News(8) | 7760.78 | 0.996 | 0.120 | 3460.38 | 0.989 |
| News(9) | 7621.81 | 0.996 | 0.120 | 3343.30 | 0.989 |
| News(10) | 7588.80 | 0.996 | 0.119 | 3259.96 | 0.988 |

## III. WATERMARK DETECTION AND EXTRACTION

By utilizing Cauchy distribution to model the DCT coefficients of H.264/AVC, the detection and extraction of bipolar additive watermark from video stream is discussed in this section. Let $t[n]$ be host DCT coefficients and $x[n]$ be watermarked coefficients. The embedding rule for an additive watermark with strength $\theta$ is given by

$$x[n]=t[n]+\theta w[n] \qquad n=0,1\cdots N \qquad (3)$$

Supposing the host coefficients $t[n]$ are random variables which follow Cauchy distribution with parameter $a=0$, it can be denoted as $t\sim C(\lambda_w,0)$. Watermark detection is in fact a detection process of signal with unknown amplitude (i.e. our watermark) from Cauchy distributed noise (i.e. the DCT coefficients)[6]. Following the hypothesis test theory, the detection of bipolar watermark can be formulated as a three-sided parameter test. The hypothesis of the parameter test are given by

$$\begin{aligned} &H_0: x[n]=t[n]\text{, without watermark}\\ &H_1: x[n]=t[n]+\theta\text{, with watermark bit +1} \qquad (4)\\ &H_2: x[n]=t[n]-\theta\text{, with watermark bit -1}\end{aligned}$$

Since the prior probabilities of these three hypotheses are unknown, Bayes decision rule is utilized to estimate $\lambda_w$ by MLE. The PDFs of $H_0$, $H_1$ and $H_2$ are defined as follows:

$$p(x|H_0)=\frac{1}{\pi}\frac{\lambda_w}{\lambda_w^2+x^2} \qquad (5)$$

$$p(x|H_1)=\frac{1}{\pi}\frac{\lambda_w}{\lambda_w^2+(x+\theta)^2} \qquad (6)$$

$$p(x|H_2)=\frac{1}{\pi}\frac{\lambda_w}{\lambda_w^2+(x-\theta)^2} \qquad (7)$$

According to the criteria of maximum likelihood, if every DCT coefficient is computed as equation (5)-(7), the decision for multiple hypotheses testing can be made by maximizing

$$\max p(x|H_i) \qquad (8)$$

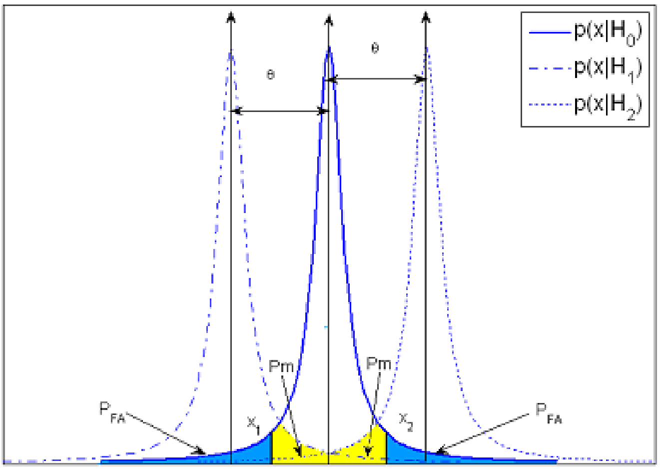


**Fig.2**. the PDFs of $H_0$, $H_1$ and $H_2$

Fig.2 illustrates the PDFs of $H_0$, $H_1$ and $H_2$. Let $P_D$ be the desired probability of true detection, $P_{FA}$ be the probability of false alarm, and $P_m$ be the probability of missed alarm. As Fig.2 shown, both $P_m$ and $P_{FA}$ are inevitable, but they can be compromised by appropriate selection of thresholds. Let $x_1$ and $x_2$ are the thresholds for $H_1$ or $H_2$, respectively, they should meet $x_1 = -x_2 = -a$. It can be found that the decrease of $P_{FA}$ is at the sacrifice of $P_m$ increase. It is impossible to decrease two kinds of error probabilities simultaneously. The design of optimal watermark detector can be achieved by appropriate selection of threshold to maximize $P_D$ which maintaining the constraint $P_{FA} = \delta$ [11]. The definitions of $P_{FA}$, $P_D$ and $P_m$ can be further derived as follows:

$$P_{FA} = \int_a^{+\infty} \frac{1}{\pi} \frac{\lambda_w}{\lambda_w^2 + x^2} dx = \frac{1}{2} - \frac{1}{\pi} \arctan(\frac{a}{\lambda_w})$$

$$P_D = \int_a^{+\infty} \frac{1}{\pi} \frac{\lambda_w}{\lambda_w^2 + (x-\theta)^2} dx = \frac{1}{2} - \frac{1}{\pi} \arctan(\frac{a-\theta}{\lambda_w})$$

$$P_m \approx \int_0^a \frac{1}{\pi} \frac{\lambda_w}{\lambda_w^2 + (x-\theta)^2} dx = \frac{1}{\pi} [\arctan(\frac{a-\theta}{\lambda_w}) + \arctan(\frac{\theta}{\lambda_w})]$$

For a given $P_{FA}$, the thresholds $x_1$ and $x_2$ are derived as:

$$x_1 = -\lambda_w \tan[\pi(\frac{1-2P_{FA}}{2})], x_2 = \lambda_w \tan[\pi(\frac{1-2P_{FA}}{2})] \quad (9)$$

For a given $P_{FA} = \delta$, whose PDF is the largest should be made according to equation (8) among the three hypothesis $H_0$, $H_1$ and $H_2$. When the DCT coefficient is less than $x_1$, $p(x|H_1)$ will be the largest. When the DCT coefficient is between the thresholds of $x_1$ and $x_2$, $p(x|H_0)$ will be the largest. When the DCT coefficient is less than $x_2$, $p(x|H_2)$ will be the largest. Thus, the watermark detection problem is converted into the decision of relationship between the DCT coefficient and thresholds. Let $x_i$ be the DCT coefficient to be detected, if it is less than $x_1$, the watermark bit will be -1. If $x_i$ is between $x_1$ and $x_2$, there is no watermark bit. If $x_i$ is larger than $x_2$, the watermark bit will be +1.

## IV. EXPERIMENTAL RESULTS AND DISCUSSION

### A. *Experimental conditions*

Four typical video sequences are encoded with JM8.6 reference code (QCIF, frame rate=30, GOP=IPPPP, QP=16). The latest video watermarking algorithms in [11] are used for performance evaluation. It employs Watson visual model for 4x4 DCT block to obtain a high payload and robustness while minimizing visual distortion. The watermark strength is set as 3 and 5, respectively. Experimental results of watermark embedding are summarized in Table II. The PSNR decrease is less that 5% and bit rate increase is about 8% on average. Though the watermark strength is relative high, watermark capacity is still acceptable and satisfactory transparency is still guaranteed.

TABLE II. PERFORMANCE OF WATERMARK EMBEDDING [11]

| Video sequences | $\theta$ | Watermark capacity | PSNR decrease | bitrate increase |
|---|---|---|---|---|
| News | 3 | 265 | 2.48% | 8.36% |
| | 5 | 265 | 3.15% | 10.27% |
| Foreman | 3 | 160 | 1.92% | 3.69% |
| | 5 | 160 | 2.30% | 6.14% |
| Akiyo | 3 | 124 | 1.55% | 7.50% |
| | 5 | 124 | 2.22% | 9.86% |
| Mother | 3 | 86 | 1.03% | 5.52% |
| | 5 | 86 | 1.58% | 8.24% |

### B. *Additive watermark detection*

The watermark detection and extraction are performed at the decoder. For a specified $P_F$, the thresholds for $x_1$ and $x_2$ will be $a$ and $-a$ respectively when the watermark strength $\theta$ is 3. By making comparison between the decoded DCT coefficients and these thresholds, the watermark bit can be obtained. The experimental results of watermark detection and extraction are summarized in Table III. If $P_F$ equals $10^{-2}$, it can detect relatively more watermark bits. However, due to the relatively low thresholds, some of DCT coefficients without watermark will be regarded as with watermark. If $P_F$ is decreased to $10^{-3}$, the correctly detected watermark bits will be less than that when $P_F$ equals $10^{-2}$. However, due to a relatively low $P_F$, the precision of correct detection still can be kept more than 83%.

TABLE III. WATERMARK DETECTION RESULTS

| sequence | $\theta$ | $P_D$ | $P_F$ | $a$ | * | ** | *** |
|---|---|---|---|---|---|---|---|
| News | 3 | 93.81% | 1.6% | 2.3893 | 265 | 247 | 86.73% |
| | 5 | 93.18% | 0.86% | 4.4479 | 265 | 206 | 89.58% |
| Foreman | 3 | 96.17% | 1.9% | 2.0008 | 160 | 161 | 91.36% |
| | 5 | 93.28% | 0.86% | 4.4405 | 160 | 147 | 89.31% |
| Akiyo | 3 | 96.08% | 1.7% | 2.0951 | 124 | 122 | 84.77% |
| | 5 | 93.08% | 0.85% | 4.1932 | 124 | 109 | 86.44% |
| Mother | 3 | 97.01% | 1.86% | 3.0125 | 86 | 91 | 83.46% |
| | 5 | 92.56% | 0.75% | 5.1364 | 86 | 82 | 86.73% |

Note: *: Watermark capacity per frame (bits); **: Correctly detected watermark bits; ***: Ratio of correct detection

An important threshold for watermark detection is $a$, which is defined as follows:

$$a = \lambda_w \tan[\pi(\frac{1-2P_{FA}}{2})] \quad (10)$$

where $P_{FA}$ is the desired false alarm probability and $\lambda_w$ is obtained by MLE according to equation (1). By experiments, we find that threshold $a$ varies between 2 and 7 for most video sequences. If we set a upper and lower limit for $a$ and make it vary between them, the watermark detection can be realized without the computation of $\lambda_w$ from the original video stream.

### C. Analysis of detector performance

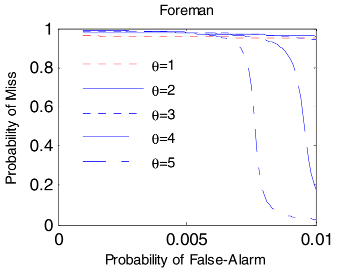
In the above experiment, satisfactory detection results are obtained when the watermark strength is 3 and 5. Generally, the transparency of video watermarking requires that the watermark strength should be low. Obviously, if the watermark strength is high, the performance of watermark detection will be more satisfactory. Therefore, it seems to be contradictory to the general requirements of watermarking algorithm.

However, the proposed watermark detection approach is still meaningful. The reasons are as follows: for most video watermarking algorithms, the watermark capacity is quite high due to its temporal redundancy. By making compromise between watermark capacity and strength, the watermark transparency can still be guaranteed. Moreover, the increase of watermark strength is beneficial to the copyright protection of digital video because the watermark will be more difficult to be removed. For example, the watermarking algorithm in [2] can be designed as follows since it is of high watermark capacity.

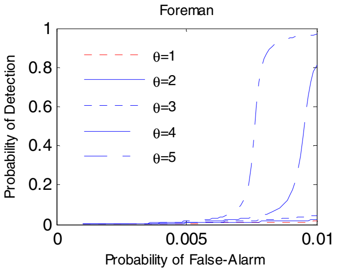
$\theta = 3$ , when $P_{FA} = 10^{-1}$

$\theta = 5$ , when $P_{FA} = 10^{-2} \sim 10^{-1}$

In Fig.3, it can found that when $P_{FA} = 10^{-1}$ and $\theta = 3$, the detector achieves satisfactory performance with a quite higher $P_D$ and quite lower $P_m$ . When $P_{FA}$ is further lowered, the response of $P_D - P_{FA}$ increases quite rapidly and the response of $P_m - P_{FA}$ decreases quite acutely, and the desired $P_D$ and $P_m$ can be obtained.



(a) Relationship between $P_{FA}$ and $P_m$



(b) Relationship between $P_{FA}$ and $P_D$

**Fig.3**. the relationship between $P_{FA}$, $P_D$ and $P_m$

## V. CONCLUSIONS

In this paper, it is verified by non-parametric hypothesis test that for those intra-coded DCT coefficients in H.264/AVC, Cauchy is better than GGD to model their statistical distribution. Then, ternary hypothesis test is introduced into bipolar additive watermark detection. This kind of ternary hypothesis testing based watermark detection is effective. Theoretically, no matter what kinds of video watermarking algorithm, if it is embedded with bipolar additive watermark with mean 0 and variance 1 and the watermark strength is reasonably adjusted, the proposed approach can realize satisfactory watermark detection. In fact, it is a statistical analysis to model the DCT coefficients with Cauchy distribution, and there will be some detection errors because of those singular samples. Experimental results show that for bipolar additive watermark in H.264/AVC, the proposed approach can achieve an accuracy of more than 80% on average for watermark detection.

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