

## BSc/MMath Projects and Literature Reviews for 2009/2010

### Department of Mathematics

The following projects and literature reviews are available for the coming academic year.

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**Supervisor: Philip Aston**

#### Dynamics of a superball bouncing on an inclined plane (project)

The bounce of a superball can be described in terms of two coefficients of restitution, the usual vertical one which incorporates the loss of energy vertically in the bounce, together with a horizontal one, where the point of contact of the ball with the surface reverses direction. This latter property accounts for the interesting behaviour that a superball can exhibit. Following on from earlier work which considered the bouncing motion of a superball on a flat plane, this project will consider the bouncing motion on an inclined plane.

#### Mathematics in Nature (literature review)

Many aspects of the natural world can be analysed or predicted using mathematics, from the motion of the planets to a prediction of the maximum height of a tree. For this review, the student should choose a variety of such topics to write about.

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**Supervisor: Michele Bartuccelli**

Michele offers projects and literature reviews in the theory and applications of ordinary and partial differential equations, mathematical fluid dynamics, Lagrangian and Hamiltonian dynamics. If you are interested please come and see me for a chat and we will try to find something that might be interesting and enjoyable for you.

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**Supervisor: Jonathan Bevan**

#### Riemann's Localization Theorem and Fourier series (project/literature review)

Riemann's Localization Theorem (RLT) is the main result behind the convergence theorems for Fourier series of sufficiently 'well-behaved' functions. For example, it is used to show that the partial sums of the Fourier series of a piecewise continuously differentiable function  $f$  converge to  $f(x)$  whenever  $f$  is continuous at  $x$ , and to  $\frac{1}{2}(f(x+) + f(x-))$  otherwise. It can also be used to infer whether the convergence is pointwise or uniform. (Students of MAT2011 Linear Partial Differential Equations will know this result well!) This literature review will explore the Lebesgue integral background to the RLT, the proof of the RLT and its applications. An advanced version could culminate in a study of Kaplansky's example of an integrable function whose Fourier series diverges everywhere.

#### Harnack's Inequality for elliptic and parabolic PDE (literature review)

In its simplest form Harnack's inequality states that if  $u \geq 0$  is harmonic (i.e., satisfies Laplace's equation) in  $B(0, R)$  then there is a constant  $C$  depending on  $R$  and the space dimension of the

problem such that

$$\sup_{B(0,R)} u \leq C \inf_{B(0,R)} u.$$

Here,  $B(0, R)$  is the open ball of radius  $R$  in  $\mathbb{R}^n$ . Various reformulations of this inequality are possible, most of which have applications in the study of elliptic PDE (like Laplace's equation) and parabolic PDE (like the heat equation). One example is the fact that if a function  $u$  is harmonic on  $\mathbb{R}^n$  and bounded below by a constant then it is identically constant, — a one-sided version of Liouville's theorem. This literature review would look at the different proofs of Harnack-type inequalities for elliptic and parabolic PDE, focusing initially on a new, direct technique in the case of ODE Harnack inequalities, and later moving on to review other applications in analysis and differential geometry.

### Solving the Dirichlet Problem: Perron's method of Subharmonic functions ([literature review](#))

The Dirichlet problem on a domain  $D \subset \mathbb{R}^n$  is

$$(P) \begin{cases} \Delta u(x) = 0 & x \in D \\ u(x) = \varphi(x) & x \in \partial D, \end{cases}$$

where  $\varphi$  is a given continuous function and  $\Delta u = \sum_{j=1}^n \frac{\partial^2 u}{\partial x_j^2}$ . Perron realised that, provided the boundary  $\partial D$  is sufficiently regular, solutions to (P) can be constructed using only continuous subharmonic functions. By definition, a *smooth* subharmonic function satisfies  $\Delta u \geq 0$  on  $D$ . But  $\Delta u$  doesn't make sense for merely continuous functions (how do you compute the derivatives, for example?), so Perron extended the notion of subharmonicity by saying that  $v$  is subharmonic on  $D$  if for every ball  $B \subset D$  (contained in  $D$  in a special sense) and every harmonic  $h$  satisfying  $h \geq v$  on  $\partial B$  it is the case that  $h \geq v$  in  $B$ . This definition, and the method devised by Perron to exploit it, has far-reaching consequences. For example, it is the forerunner of the modern viscosity solution methods for solving 'difficult' nonlinear equations such as the Monge-Ampère equation. This literature review will examine Perron's method in detail, culminating in a study of Wiener's criterion for the regularity of  $\partial D$  and selected examples of the viscosity solution method in action.

### 'Weak' Differential Equations ([literature review](#))

The Picard-Lindelöf Theorem states that solutions to

$$y'(x) = f(x, y(x))$$

exist and are unique (subject to appropriate initial data) provided  $x \mapsto f(x, \cdot)$  is Lipschitz continuous for each fixed  $x$  and  $w \mapsto f(\cdot, w)$  is continuous for each fixed  $w$ . (Paraphrasing a bit here.) If  $f$  is assumed to be merely continuous (a weaker condition than being Lipschitz) then Peano's Theorem asserts that solutions exist but that they may not be unique subject to initial data. What happens when  $f$  is not even assumed to be continuous? Carathéodory showed us that if we are prepared to weaken our notion of solution then existence can still be recovered under quite mild hypotheses on the function  $f$ . These three existence theorems form the starting point for a literature review of what is meant by a 'weak solution' of a differential equation. It will eventually lead to the study of absolutely continuous functions and applications to the Euler-Lagrange equations of the Calculus of Variations.

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**Supervisor: Henk Bruin**

**Literature reviews** in:

- **Dynamical systems**, such as
  - Sharkovskii's Theorem.

- Symbolic dynamics and Li-Yorke Chaos.
- Renormalisations of quadratic maps (Feigenbaum maps and its relatives).
- Complex dynamics (Julia sets, the Mandelbrot set)
- **Number theory**, such as
  - Continued fractions and Farey trees.
  - Properties of binomial coefficients.
- **Topology**, such as
  - The four colour problem.
  - Space filling curves.
- **History of Mathematics**, for example
  - the work of Blaise Pascal (1623 - 1662), one of the founding ‘fathers’ of probability, but also active in mathematical physics,
  - or less famous mathematicians who have made some quite distinct contribution, e.g. John Wallis (his product), James Stirling (his formula), Caspar Wessel (complex numbers), or Srinivasa Ramanujan (Number theory).

### Computation of Hubbard Trees: the skeleton of the Julia set (project)

Quadratic iterations is a complicated, chaotic process, and even more so when performed on the complex plane. It gives rise to Julia sets and the Mandelbrot set, which is considered by some the most intricate mathematical shape known today. To understand Julia sets theoretically, there are methods available from coding theory as well as a thing called Hubbard tree. This is the “abstract skeleton” of the Julia sets, and it encodes information about e.g. the branchpoints in “lightning-bolt-shaped” Julia sets. If you are familiar with Java, you could choose this project to implement (and improve?) existing algorithms to visualise Hubbard trees.

### Supervisor: Jonathan Deane

Jonathan Dean is willing to supervise general computing projects based on your own ideas: come and talk to me if you have an idea you want to discuss.

### Chebyshev's Theorem and Chebyshev polynomials (project/ literature review)

Chebyshev's theorem states: Let  $p(x)$  be a polynomial of degree  $n \geq 1$  with leading coefficient (i.e. coefficient of  $x^n$ ) = 1. Then the maximum value of  $|p(x)|$  over the range  $x \in [-1, 1]$  is  $\geq 1/2^{n-1}$ . You will first need to understand a proof of this theorem. There is a set of orthogonal polynomials, known as the Chebyshev polynomials, for which the maximum value exactly equals  $1/2^{n-1}$ , and these polynomials have many other interesting properties as well, a selection of which you will investigate.

### Probabilistic Primality Testing (literature review)

It is possible to test whether a number  $N$  is prime or not by testing whether it is divisible by any of the primes less than square root of  $N$ . For large numbers, this obviously takes an inordinate amount of time. There are many other tests that can be used, and one at least involves probabilistic methods that gives an answer with a probability close to, but not equal to 1. To find out how this is done, the methods limitations, and why primality testing is important in practice (e.g. in relation

to “uncrackable” codes) is the subject of this literature review.

### Teaching software for the World Wide Web (ONE project only)

If you know about at least one of html, Java, cgi you might consider writing some teaching software for the Web. For a good example of previous projects of this type, see Nigel Martins Web pages at [<http://www.geocities.com/nigelpm/html.htm>]. There is a lot of scope for good Web page design and there is a free choice about the area of the course you choose to cover.

*Standard:* Anything from easy to difficult. The perfect example of an elastic project.

### Worldwide accessible computation software (project)

It is desired to make available via the World Wide Web a program that does a numerical evaluation of an integral, for given sets of input parameters. The program is already written; the project is to interface this with the WWW, possibly by machine translating it into Java or in any other practicable way. Knowledge of WWW technologies required.

### An analysis of ‘Aces up (project)

“Aces up is a very simple game of patience, using a single standard pack of cards. The rules can be found at [<http://www.kmgassociates.com/games/acesup/>]. Experience (i.e. playing it a lot) gives the impression that the probability of succeeding at the game is something of the order of 1/50. The objective of this project is to estimate this probability by Monte Carlo simulation, and determine whether there is any strategy that can increase this probability.

*Standard:* Needs good programming skills and a little bit of elementary probability theory. Moderate standard.

### An analysis of ‘Paradoxical Pennies (project)

An amusing game based on probability. In the original version, your opponent is asked to choose a sequence of three heads and tails, e.g. HTH; you then choose a sequence, and toss a coin repeatedly until one or other of the sequences comes up. The winner is the person whose sequence comes up first, and you can always choose a sequence that has a probability of at least 2:1 of coming up first. How to do this is described in ‘Winning Ways’, volume 2, by Conway, Berlekamp and Guy (its in the library). A program to estimate the probabilities for the game played with sequences of length  $n$ , consisting of  $m$  symbols ( $n = 3, m = 2$  in the original version) is required. A nice interface, possibly in Java would be a useful extra.

*Standard:* Moderately simple if you are already a competent programmer.

### Finding poles in the complex time domain solution of Duffing’s equation (project)

Duffing’s equation:

$$\frac{d^2y}{dz^2} + k \frac{dy}{dz} + By(y^2 - 1) = A \sin(\omega z),$$

is a well-known nonlinear differential equation whose solutions (for real time,  $z = t$ ) display chaotic behaviour for some values of the parameters. Using a standard numerical differential equation solver from the NAG library, the objective is to write a program to solve this equation for complex time,  $z$ . There are poles off the real axis in the solution of the equation, which are (a) movable (their position depends on the initial conditions) and (b) conjectured to be simple, i.e. of the form  $a/(z - z_1)$ . The project will be to write software to locate some of these poles and find their orders and residues using a ratio test. The challenging part of the project is to carry this out as automatically as possible – that is, without human intervention.

*Standard:* Very challenging and interesting project. Needs programming skills and understanding of

some of the topics covered in the Level 2 Complex Variable course (the Cauchy-Riemann conditions and Cauchy-Goursat theorem for instance).

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## Supervisor: **Gianne Derks**

Apart from the projects/literature reviews suggested, more ideas can be found on Gianne's webpage: [<http://personal.maths.surrey.ac.uk/st/G.Derks/teaching.php>]

If you have some other ideas for a project or literature review, feel free to come and discuss those.

### Can you hear the shape of a drum? (project/literature review)

"Can you hear the shape of a drum?" is the title of an article by Mark Kac in the American Mathematical Monthly 1966. The sound of a drum is associated with its harmonics. By using Helmholtz equation, the harmonics can be determined if one knows the shape of a drum. And the question of hearing the shape of a drum is asking if the shape of a drum can be found if all harmonics are known. This project/literature review can take several directions. You can look at how the harmonics can be found if the shape is known. And/or you can look at the question of hearing the shape of a drum. In the beginning of the nineties, it is shown that the answer is "no". But you need a non-convex drum. For certain types of convex drums, it can be shown that the harmonics are unique.

### Stationary solutions in materials with defects (project)

The background for this project is in superconducting materials, called Josephson junctions. But you don't have to know about anything about superconducting materials or physics to do this project. The project focuses on a mathematical model for the dynamics in the superconducting material. The model is well-known and called the sine-Gordon equation. This project will look at a modified sine-Gordon equation to model the presence of defects, i.e., (small) scratches, imperfections, etc., and focus on changes in stationary solutions for model. Although this model comes from physics, similar issues come up in other models, like ones from biology, ecology, economics, etc. The introduction above might sound complicated, but the ideas for the project are quite simple and a lot of fun. A few ideas of ODEs will be used, but there will be a lot of different ideas as well. One of the techniques is matching of two of three phase portraits, such that they nicely align at the defects. This is a simple, but very efficient technique. The project will teach you about those techniques. The project is fairly flexible and can vary from mainly analytical to numerical (with MATLAB).

### Bessel functions and Navier-Stokes equations (project/literature review)

The solutions of the differential equation  $u''(x) = -k^2u(x)$ , with  $k$  some constant in  $\mathbb{R}$ , are linear combinations of the functions  $\sin kx$  and  $\cos kx$ . Boundary conditions, for example  $u(0) = 0 = (1)$ , select a subset of the total set of solutions and usually add a condition on  $k$ , in this case  $k = n\pi$ ,  $n \in \mathbb{N}$ . On a square, something similar happens: the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -(k^2 + l^2)u,$$

has as solutions linear combinations of  $\sin kx \sin ly$ ,  $\sin kx \cos ly$ ,  $\cos kx \sin ly$  and  $\cos kx \cos ly$ . Boundary conditions on the sides of the square will give conditions for  $k$  and  $l$ . However, if we consider the equation on a circular disk, the sine and cosine functions are not very convenient to deal with any boundary conditions on the disk. In this case, the so called Bessel functions are more convenient. These functions are well-studied and have very nice properties. For this project or literature review, you will first read through literature to get an overview of some of the properties of Bessel functions. For the project you will continue to apply the Bessel functions to analyse certain solutions of the Navier-Stokes equations. The Navier-Stokes equations describe the motion of fluids. They are quite

famous, you can even earn one million dollar by proving that solutions do (not) exist and are (not) uniqueness for the three dimensional version, see

[[http://www.claymath.org/millennium/Navier-Stokes\\_Equations/](http://www.claymath.org/millennium/Navier-Stokes_Equations/)].

This project is less ambitious and will focus on a two-dimensional problem and the influence of boundary conditions. The Bessel functions are used to describe certain solutions. The project will involve a combination of analysis and numerics (MATLAB/MAPLE).

### **Rosby waves and the spring-swing (literature review/project)**

The large sinusoidal oscillations of the atmospheric flow are called Rossby waves. They are dominant in determining the patterns of weather and climate in middle latitudes, in particular the changeable weather with which we are blessed. You can see them any night on the TV weather forecast maps. Rossby waves interact with each other in groups of three, known as resonant triads and, for small amplitude, they are described by the three-wave equations. Their interactions are crucial for determining the distribution of energy in the atmosphere. A couple of years ago it was discovered that these same equations also govern the dynamics of a simple mechanical system, the elastic pendulum, comprising a heavy mass suspended by a spring. Thus, the motion of a swinging spring gives us information about resonant triads. For a literature review, you can investigate the connection between Rossby waves and the swinging spring. For a project you can find some of the solutions and see how solutions can change suddenly if parameters change.

### **Wavelets (literature review)**

The fundamental idea behind wavelets is to analyse according to scale. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. The idea is similar to the idea behind Fourier series and integrals where sines and cosines are used to represent other functions. However, wavelets are better in taking different scales into account. If we look at a signal with a large window, we would notice dominant features. Similarly, if we look at a signal with a small window, we would notice small features. The result in wavelet analysis is to see both the forest and the trees, so to speak. The advantage of wavelets over traditional Fourier is especially noticeable when the data or function contains discontinuities and sharp spikes. Applications of wavelets are widely varied and include astronomy, acoustics, data compression, nuclear engineering, sub-band coding, signal and image processing, neurophysiology, music, magnetic resonance imaging, speech discrimination, optics, fractals, turbulence, earthquake-prediction, radar, human vision, and pure mathematics applications such as solving partial differential equations.

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## **Supervisor: David Fisher**

David is happy to supervise projects and literature reviews in aspects of linear and abstract algebra.

### **Topics in Linear Algebra (literature review)**

Matrix and vector space methods are of interest in their own right, and are also widely used in applications such as Coding, Graph Theory, Image Processing and Optimisation. You could carry out a wide-ranging review or a detailed examination of one or two aspects, looking into the origins of the methodology, refinements and recent developments.

### **Lie Algebras (project/literature review)**

You have met several examples of algebraic structures such as vector spaces, groups, rings and fields. A Lie algebra is a vector space on which a particular kind of multiplication is defined. The aim of the review or project is to investigate how Lie algebras arise and to explore their classification into

different types. In many cases the algebra can be represented by matrices, which provides opportunities to analyse it using computer algebra.

### Animated Teaching Software (project)

Most lecture rooms are now equipped with audio-visual equipment which enables pre-prepared material to be projected from a computer. Rather than just projecting printed notes, it would be useful to have some animations to bring dynamic aspects of Mathematics to life. To do this project you will need to have the necessary programming skills and some good ideas about suitable topics to tackle. You could also look into existing material of this nature, comment on its effectiveness and consider how it could be improved.

## Supervisor: Janet Godolphin

### Construction and Uses of Sets of Mutually Orthogonal Latin Squares (project)

A Latin square of order  $n$  is an  $n \times n$  array of  $n$  symbols arranged so that each symbol occurs once in each row and once in each column. Two Latin squares,  $A$  and  $B$ , of order  $n$  are said to be mutually orthogonal if, when one square is superimposed on the other, each of the  $n^2$  ordered pairs of symbols  $(a, b)$  occurs exactly once. For example, the following Latin squares of order 5 are mutually orthogonal:

Square A					Square B				
0	1	2	3	4	0	1	2	3	4
1	2	3	4	0	2	3	4	0	1
2	3	4	0	1	4	0	1	2	3
3	4	0	1	2	1	2	3	4	0
4	0	1	2	3	3	4	0	1	2

Orthogonal Latin squares do not exist for all values of  $n$ , but for any prime power  $n$  there exist  $n - 1$  mutually orthogonal Latin squares (MOLS) of order  $n$ . The objective of this project is twofold.

- Understand and summarise the theory underlying the existence of sets of MOLS. Investigate the various methods of construction of sets of MOLS. Produce a program to generate sets of MOLS for suitable  $n$ .
- Using the program, demonstrate the relevance of MOLS in a range of situations as diverse as design of experiments, cryptography and tournament planning.

No prior programming experience is necessary.

### Semi-Latin Squares and Trojan Squares (project)

Latin squares are valuable in experimental design to produce designs that take account of two sources of heterogeneity in experimental units that may influence the response variable. Here is an example of a Latin square design, of order 4, used to investigate four industrial processes  $\alpha, \beta, \gamma$  and  $\delta$  taking account of two sources of heterogeneity, namely *machine* and *day*.

		Day			
		1	2	3	4
Machine	1	$\alpha$	$\beta$	$\gamma$	$\delta$
	2	$\beta$	$\alpha$	$\delta$	$\gamma$
	3	$\gamma$	$\delta$	$\alpha$	$\beta$
	4	$\delta$	$\gamma$	$\beta$	$\alpha$

Such *row-column* designs are particularly useful in industrial and agricultural research situations but their practical size range is limited. Semi-Latin square designs extend this range, with a semi-Latin square being defined as an arrangement of  $nk$  symbols in  $n$  rows and  $n$  columns, each row-column intersection containing  $k$  units and each symbol occurring exactly once in each row and column. A further type of design is given by Trojan squares which form a special class of semi-Latin squares that generalises the class of Latin square designs. In general, before experimentation takes place, competing experimental designs are assessed and compared via optimality criteria, namely A, D and E optimality criteria.

The objective of this project is to:

- Investigate the uses of Latin squares, semi-Latin squares and Trojan squares in experimental design and their methods of construction.
- Understand and summarise the theory behind the A, D and E optimality criteria.
- Write a program to calculate the A, D and E optimality criteria for a given row-column design.
- Using the program, evaluate the optimality criteria for semi-Latin square designs available in the literature. Demonstrate the E optimality of Trojan squares.

No prior programming experience is necessary.

### **Bootstrapping (project)**

In statistics we often wish to use a sample to make inferences about an unknown parameter,  $\theta$  say, of the population from which the sample was drawn.

Two questions of importance are:

1. What estimator of  $\theta$  should be used?
2. Having chosen an estimator, how accurate is it?

In order to answer the second question, information is needed on the distribution, or at least the variance, of the estimator. When the parameter of interest is the mean,  $\mu$ , then the variance of the estimator  $\bar{X}$  is known to be  $\sigma^2/n$ . However, if an estimate of the population median is required, for example, then no estimator with known variance exists. The technique of Bootstrapping gives a method for answering Questions (1) and (2) when all we can assume is that our data are reasonably typical of the parent population. Bootstrapping involves resampling from the initial sample many times and using the resamples to draw inferences about the corresponding population and its parameters. The Bootstrap conjecture is that the sampling distribution of parameter estimates based on resampling mirrors the sampling distribution of the parameter of interest.

The objectives of the project are:

- To understand and describe the Bootstrap technique and the underlying theory.
- To use R to perform the technique of Bootstrapping.
- To make inferences on real and simulated data sets using the Bootstrapping algorithm.

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### **Supervisor: Stephen Gouley**

Stephen offers [projects](#) and [literature reviews](#) in the theory and applications of ordinary and partial differential equations and mathematical analysis.

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**Supervisor: Rebecca Hoyle**

**Modelling in systems biology (Literature review spring semester only)**

Systems biology is the study of the complex networks of interactions in living organisms, for example, the network of chemical reactions in the metabolism of a bacterium. Mathematical modelling is very helpful in understanding these systems. Dynamics is particularly important, and tools from dynamical systems, such as bifurcation analysis, are often used. A literature review could examine any aspect of mathematical modelling in systems biology.

**Molecular motors (Literature review spring semester only)**

Molecular motors are proteins that turn the chemical energy released in ATP hydrolysis into mechanical work. Motors can walk along tracks, rotate, and pull on other molecules. For example, the molecular motor myosin II works to contract our muscles so that we can breathe, move around, pick things up, open and close our eyes, and so on. A literature review might look at the various different classes of mathematical model for molecular motors, or focus on a particular type of model in depth.

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**Supervisor: Peter Hydon**

**The KdV equation (literature review)**

The Korteweg-de Vries (KdV) equation is a third order partial differential equation with various remarkable properties. Its solutions include solitons, which are travelling waves that can interact with other waves without being destroyed. This review focuses on the huge range of solution methods that are available for KdV and other integrable partial differential equations.

**Differential forms and de Rham cohomology (literature review)**

Differential forms provide a coordinate-free framework for describing calculus on manifolds. This enables one to prove many important local theorems. (The proofs would be hopelessly complicated if coordinates were used.) The de Rham cohomology provides a means of extracting topological information about a given manifold, and using this to construct powerful theorems that are valid globally. This review is a methods-based study of the above ideas for the plane, the sphere and the torus.

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**Supervisor: David Lloyd**

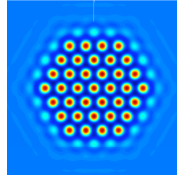
Dave is willing to supervise general computing projects based on your own ideas: come and talk to me if you have an idea you want to discuss.

**Partial differential equations - Localised patterns (project/literature review)**

Localised patterns are observed in a variety of experiments ranging from nonlinear optics, to vegetation growth in arid environments and vertically vibrated sand; see figure above.

for an example of localised patches that are observed in a PDE model. The fundamental challenge is to understand the formation and stability properties of localised patterns in 2- and 3-dimensions.

A literature review of this topic will look at localised patterns that occur in physics and the what is known about the mechanisms governing their behaviour.



Possible projects will look at developing understanding of the interaction properties of two well separated patches of cellular hexagons, nucleation of localised structures under the influence of space/time noise, effects of finite domain and oscillating localised structures; these projects would be suitable for those that like programming with MATLAB. For those that are more analytically minded, we would look at the emergence of localised radially symmetric patterns.

### **Mathematical biology - Swarming of bees (project/literature review)**



A project/literature review would look at the (mathematical) rules that govern swarming of insects, shoaling of fish and flocking of birds. You will investigate various agent-based models and, for the project, look at analysing those models using MATLAB.

### **Mathematical biology - tumour modelling (project/literature review)**

Oxygen hypoxia plays an important role in tumour resistance to radiotherapy treatments, chemotherapy and surgery. In this project/literature review, you will look at mathematical models for oxygen distribution in tumours and radio-active tracers used for radiotherapy.

### **Modelling and Simulating Chemical Reactions (project/literature review)**

Have a look at the YouTube video of a chemical oscillation: [[CHEMICAL OSCILLATIONS](#)]. In this project you will look at the differential equations governing these and other chemical reactions.

### **Chaos & Fractals (project/literature review)**

Have a look at the Chaos & Fractals website, videos and notes:

[<http://personal.maths.surrey.ac.uk/st/D.J.Lloyd/MAT3007.php>]

I am happy to supervise a project or literature review on anything topic in this course. This project/literature review would be excellent for students taking the Chaos & Fractals course. Come and see me to have a chat.

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**Supervisor: Ian Melbourne**

### Chaos in the logistic map (project)

Consider the logistic map  $f(x) = \mu x(1 - x)$ . For  $0 < \mu < 3$ , there is a unique attracting fixed point, but if a parameter  $\mu \in [3, 4]$  is chosen at random then there is a positive probability that the dynamics is chaotic. Numerical experiments suggest that the probability of chaos is quite high. Unfortunately, the best theoretical results currently available guarantee only that the probability of chaos is at least  $10^{-5000}$ .

The aim of the project is to use a new test for chaos to estimate numerically (using MATLAB or C) the probability of chaotic dynamics in the logistic map.

This project would make most sense for a student planning to take the *Chaos and Fractals* course.

### Random walks in random environments (project)

The simple symmetric random walk (SSRW) on  $\mathbf{Z}$  is considered in *Stochastic Processes* and is shown to be recurrent. It follows from the Central Limit Theorem that the growth rate (or diffusion rate) is  $\sqrt{n}$ . (Roughly speaking, if  $X_n$  is the position at time  $n$ , then  $|X_n| \sim \sqrt{n}$ .)

The SSRW is an example of a random walk in a fixed environment. In 1982, Sinaï proved the remarkable result that for a recurrent random walk in a *random* environment on  $\mathbf{Z}$ , the diffusion rate is  $(\log n)^2$ .

The aim of this project is to (i) study Sinaï diffusion numerically (using MATLAB or C), and (ii) investigate some related examples where the randomness of the environment is introduced in a reflectionally symmetric manner. In the situation of (ii), it seems plausible that the diffusion rate is neither  $\sqrt{n}$  nor  $(\log n)^2$ .

*This can be done either as a Mathematics or Statistics project.*

### Hypermeandering spirals in a spherical shell (project)

In numerical simulations of waves in planar chemical reactions, spiral waves are observed to exhibit complicated chaotic dynamics where the spiral tip appears to undergo a random walk in the plane. This phenomenon, known as hypermeander, can be explained by (i) reducing from the appropriate partial differential equation (PDE) model to a finite system of ordinary differential equations (ODEs), and (ii) analysing the asymptotic statistical properties of the reduced chaotic ODE.

The analogous situation for chemical reactions in a spherical shell is not yet understood, due to technical difficulties of a theoretical nature in step (ii). However, there is no obstruction to studying the reduced ODE numerically (using DSTOOL and/or MATLAB). Alternatively, the identical questions arise for discrete dynamical systems, so the continuous-time ODE can be replaced by a discrete-time map. The aim of this project is to obtain numerical data on the asymptotic statistics of the ODE/map and to determine whether this data supports the conjecture that the ODE/map accurately describes hypermeander in a spherical shell.

*This can be done either as a Mathematics or Statistics project.*

### Convergence of deterministic equations to stochastic equations (project)

Consider the *fast-slow* system of ODEs

$$\begin{aligned}\dot{x} &= \epsilon^{-1} f_0(y) + f(x, y) \\ \dot{y} &= \epsilon^{-2} g(y)\end{aligned}$$

where  $\epsilon$  is a small parameter and  $x \in \mathbf{R}^n$ ,  $y \in \mathbf{R}^m$  represent “slow” and “fast” variables respectively. Under mild conditions on the fast  $y$  dynamics, the solution to the  $\dot{x}$  equation converges to a solution of a stochastic differential equation as  $\epsilon \rightarrow 0$ . The randomness stems from the choice of the initial condition for  $y$ .

A natural question is to consider what happens if the  $f_0$  or  $g$  terms are allowed to depend also on  $x$ . The expectation is that a  $g(x, y)$  term should not make much difference (though it is harder to prove things) but that an  $f_0(x, y)$  term makes a significant difference.

This project would make most sense for a student planning to take the *Advanced Stochastic Modelling* course.

*This can be done either as a Mathematics or Statistics project.*

### **Symmetric chaos in bifurcations with symmetry (project)**

Symmetric chaos (the coexistence of chaotic dynamics with regular symmetry) has been shown to be widespread in a recently-studied example of a bifurcation in a system with square symmetry.

The main interest of this project is to study a related example that seems to have all the ingredients to produce large scale symmetric chaos as in the existing example, but has the advantage that it should be more readily accessible in applications (such as a chemical reaction modelled by a reaction-diffusion equation). This means starting from scratch, but the methodology is similar to the previous example.

Another possible project is to consider related bifurcations with pentagonal, hexagonal,  $\dots$ , symmetry.

These projects would make most sense for a student planning to take the *Nonlinear Patterns* course and would involve analytic calculations together with numerical study of differential equations using DS TOOL and/or MATLAB.

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## **Supervisor: Mark Roberts**

### **Graphs and Groups (project)**

A *graph* is a collection of points (vertices) and line segments (edges) joining pairs of vertices. A *path* in a graph is a sequence of edges. A *symmetric graph* is one on which a group acts by permuting the vertices in such a way that edges are taken to edges. Paths in a symmetric graph can have different degrees of symmetry depending on which group elements map the paths to themselves. The aim of this project is to develop a method for counting the number of paths of a given symmetry type joining two given vertices. Apart from elementary graph theory the main ingredients will be some linear algebra and group theory. This project would be suitable for somebody who has taken the level 2 module Groups and Symmetry and who intends to take the level 3 module Nonlinear Patterns.

### **Convex Coverings of Polytopes (project)**

A *polygon* is a region in the plane bounded by a finite number of straight line segments. A polygon is *convex* if the straight line segment between any two points in the region also lies in the region. Every polygon can be written as the union of a finite number of convex polygons (exercise: prove this). The first aim will be to obtain an upper bound on the smallest number of convex regions needed to cover any  $N$ -sided polygon and to relate this number to the topology of the region. We will then consider generalisations to higher dimensional *polytopes*.

### **Configuration Spaces and Control of Linkages (project/literature review)**

A *linkage* is a ‘machine’ consisting of several rigid rods connected by joints. The angles between the rods can be varied by rotating the joints. The *configuration space* of the linkage is the set of all possible ‘shapes’ that it can form. For an introduction see, for example:

[<http://www.math.toronto.edu/~drorbn/People/Eldar/thesis/>].

A literature review could describe aspects of the topology of these configuration spaces, or the kinematics and dynamics of their motions. Aims for a more extended project might include exploring how to move the linkage from one configuration to another ‘as fast as possible’.

### Examples of Rigid Body Dynamics (project/literature review)

The aim of a project or literature review in this area will be to study in some detail the dynamics, and possible control, of a particular example of rigid body dynamics. Possibilities include:

- Spinning tops;
- A rigid satellite orbiting the Earth;
- Binary asteroid systems;
- A rigid body in a fluid;
- Two or more rigid bodies in a fluid;
- A charged rigid molecule in an electric and/or magnetic field;
- Two charged rigid molecules interacting with each other.

Students working on these projects will also need to take the level 3 module Lagrangian and Hamiltonian Mechanics.

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### Supervisor: Ian Roulestone/Andy White

Andy White is a visiting professor from the Met office and maybe contacted via email: andy.white@metoffice.gov.uk.

### Meteorology & Geophysical Fluid Dynamics (project/literature review)

The following list of 15 possible projects contains 12 that are regarded as effectively independent, main projects, and three that may usefully be considered as developments from them (in fact, from only two of them).

The 12 main projects divide readily into four sets:

- (F) Fundamental studies of the equations governing rotating fluid motion
- (DP) Derivation/presentation of specific approximate versions of these equations
- (EC) Evaluation and critique of specific approximate versions
- (N) Studies relating to discretizations for use in numerical models (N).

The DP set contains only one member, and it is the first member of the list. The three projects that are developments of main projects are denoted (with some redundancy) as Dev.

In contemplating the list, it should be remembered that approximate versions of the governing equations are the basis of most of the numerical models used in meteorology, climatology and oceanography for forecasting and simulation, and of almost all analytical studies in these subjects.

Projects 1 - 12

1. Coordinate-independent differentiation (via the exterior calculus) and its possible application to deriving the properties of approximate models that incorporate either or both of the shallow-atmosphere and hydrostatic approximations. (DP)

2. Trajectory-based presentations of the equations of motion in 2 space dimensions (in terms of flow speed and direction, and natural coordinates). (F)
3. Trajectory-based presentations of the equations of motion in 3 space dimensions (in terms of the geometry of twisted curves and quantities such as spin vectors). (F)
4. Consequences (for approximate models) of the dependence of the conventional definition of horizontal on the chosen rotating coordinate frame. (F)
5. Evaluation/critique of the use of pressure as vertical coordinate in nonhydrostatic models. (EC)
6. Evaluation/critique of the shallow water equations as a model of large-scale atmospheric flow. (EC)
7. Evaluation/critique of the nonlinear balance equation and its use in meteorology (especially in the assimilation of data into numerical forecasting models). (EC)
8. Critique of balanced approximate models that lack precise analogues of potential vorticity conservation (including zonally-averaged models). (EC)
9. Appraisal of the filtering of internal gravity waves from balanced models, and justification of it in terms of timescale separation or otherwise (if possible). (EC)
  - (a) Development of partially-balanced approximate models in which only the height-averaged flow is balanced and internal gravity wave propagation is allowed. (Dev)
10. Updated justification, derivation and evaluation/critique of the classical quasi-geostrophic (QG) model - given recent developments in asymptotic analysis. (EC)
  - (a) Development of models of QG type for flow over intrusive orography. (Dev)
  - (b) Development of models of QG type for flow on the sphere when horizontal variations of buoyancy frequency are not small. (Dev)
11. The fate of conservation properties and stability criteria under various numerical discretizations of the continuous governing equations. (N)
12. The treatment of equatorial modes of motion (such as Kelvin waves) by various numerical discretization schemes. (N)

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## Supervisor: Peter Williams

If you are interested in any of the following statistical projects or reviews, please contact:

P.Williams@surrey.ac.uk

### The Multiple Comparisons Testing issue (**literature review**)

Research projects often result in large numbers of significance tests, especially if there are many subgroups to be compared. Statistical output can be difficult to interpret properly owing to the fact that at the standard significance level of 5% (i.e.  $p < 0.05$ ) we expect to encounter 1 in 20 statistically significant results purely by chance.

Various adjustments, known as multiple comparison tests, have been utilised over the years to counter this effect, such as the Bonferroni Method, the Tukey-Kramer test, Fisher's LSD test and Scheffe's test. There are also procedures known as multiple range tests, such as the Newman-Keuls test and the Duncan Multiple Range test. This literature review will focus on these latter tests.

After determining the mathematics behind each multiple range test, a study will be carried out of recent discussions in refereed journals (e.g. the BMJ), along with the identification of recent papers

which have utilised multiple range tests.

### **Assessment of inter-rater reliability under various scenarios (literature review)**

Various measurement techniques exist for assessing the degree of agreement between  $n$  raters classifying the same sample of subjects. The techniques are dependent upon whether  $n = 2$  or  $n > 2$ , and also upon the nature of the classification variable, which can be dichotomous (e.g. Yes or No, Accept or Reject), interval (e.g. a score out of 10, a VAS score out of 100) or ordinal (e.g. 1=Strongly Agree 2=Agree 3=Neutral 4=Disagree 5=Strongly Disagree, 1=Yes 2=Unsure 3=No).

Additionally, one or more of the raters may be regarded as the gold standard. Furthermore, a test-retest situation, where the same raters classify the same subjects on more than one occasion, may arise.

The need for assessing raters occurs in many fields, including essential ones (e.g. medical: assessing nurses in the diagnosing of a disease) and sporting ones (e.g. gymnastics/ice skating: assessing international judges in the scoring of Olympic contestants).

A literature search of statistical, medical and other textbooks/journals is needed in order to assemble a grid containing the most useful technique(s?) for each possible scenario.

For example, in the case of 2 raters rating subjects for an ordinal classification variable, the simple Kappa statistic can be calculated from a crosstabulation of their ratings. However the weighted Kappa statistic, which incorporates the extent of any disagreement between the 2 raters on any subject, clearly has more desirable properties. See *Statistical Methods for Rates and Proportions - Third Edition*, by Fleiss, Levin and Paik (Chapter 18) for further details.

For a relatively recent approach when continuous (interval) data are involved, see *Statistical methods for assessing agreement between two methods of clinical measurement* by Martin Bland and Douglas Altman (The Lancet, 1986, pages 307-310).

Note that for a student with good statistical computing skills, this literature review could be expanded into a project.

### **The derivation and usage of trend tests (literature review)**

Not a day goes by without a newspaper headline informing us that some entity (whether it be a disease, a sport related outcome, the "cost of living", etc.) is increasing or decreasing. Such headlines are often derived from academic journals, where (hopefully) an appropriate statistical test has been performed in order to reach this conclusion. Ignoring the trivial case where only two time points have been considered, a literature review is needed to identify which trend tests are being used, and to determine whether certain industries or publications favour particular trend tests.

The two prime candidates for proportion data (i.e. percentage of all cases with a specific trait) over 3 or more time points are the Chi-squared trend test (as recommended by Altman, "Practical Statistics for Medical Research", Chapman & Hall (1991)) and the Cochran-Armitage trend test (Armitage, "Tests for linear trends in proportions and frequencies", *Biometrics* (1955) 11:375-386). The origin and usage of these tests need to be fully documented.

A similar process should be undertaken for continuous data (e.g. house prices).

The issue of the effect of non-random missing data at various time points might also be considered, should somebody with good statistical programming skills wish to convert this proposed literature review into a project.

### **The consistency and robustness of goodness-of-fit statistics for logistic regression models (project)**

Whereas the R-squared statistic provides an unequivocal indicator of the goodness-of-fit of a linear regression model, no such statistic exists for a logistic regression model (i.e. when modelling a dichotomous outcome variable).

There exist subjective techniques involving the inspection of graphs, along with several single value statistics (e.g. Cox & Snell, Nagelkerke) and the Hosmer & Lemeshow approach. And in some studies it is possible to assess the goodness-of-fit by adopting a test & validation set approach, where a subgroup of the data is used to construct the model and the remaining data are then used to determine the accuracy of the model in real terms (i.e. according to whether each outcome in the validation set was correctly predicted or not).

Two recent medical databases (one with a small sample size, one with a large sample size) are available to assess goodness-of-fit statistics. Good statistical programming skills will be needed.

Since this project forms part of an ongoing medical research study, for planning purposes please contact me as soon as possible to express an interest (no need to wait until Options Day).

### **Ordinal Logistic Regression (project)**

Whilst the modelling of a continuous dependent variable (e.g. a test score on a scale of 0 to 100) against a set of independent variables is performed using a standard technique, i.e. least squares multiple linear regression, the optimal technique for modelling an ordinal dependent variable (e.g. health = Excellent/Good/Fair/Poor) is not as clear cut.

The 4 most well known techniques are listed below. See the book "An introduction to GLMs" by Annette J Dobson (Chapman & Hall 2002) for a brief overview of these techniques.

1. The Proportional Odds Model
2. The Cumulative Logit Model
3. The Continuation Ratio Model
4. The Adjacent Categories Model

See also "Regression Models for Ordinal Responses: A Review of Methods and Applications" by Ananth and Kleinbaum (International Journal of Epidemiology, 1997, Vol.26, 1323-1333) for a more extensive overview.

Selecting specific techniques, in-depth analyses of a large medical data set (to be provided) are needed in order to assess regression diagnostics, notably goodness-of-fit indicators. Some statistical programming skills will be needed.

Since this project forms part of an ongoing medical research study, for planning purposes please contact me as soon as possible to express an interest (no need to wait until Options Day).

### **An investigation into the possible impact of carry-over effects in cross-over trials (project)**

Opinion differs as to how to identify and deal with carry-over effects in cross-over trials. Gerard Dallal [[www.jerrydallal.com](http://www.jerrydallal.com)] insists that data from subsequent treatment periods should be discarded if carry-over effect is identified, which appears to be the standard procedure within the pharmaceutical industry. Others (e.g. Stephen Senn) suggest running appropriate ANOVAs to adjust for any carry-over effect.

Various underlying distributions for outcome variables in such scenarios need to be explored in order to assess the implications of different decisions regarding carry-over effects. Distributions should include those for the 'validated questionnaire theme means' family, where the same questionnaire is used at different timepoints to assess an outcome via the mean of several 'Strongly disagree/Disagree/Neutral/Agree/Strongly agree' questions, thus involving a 'squeezed' (between scores of 1 and 5) derived mean with a subject's ability to score higher/lower being compromised if their baseline score is different at the start of each treatment.

Recommended reading: Cross-over trials in Clinical Research by Stephen Senn (Wiley 1993)

## **Statistical techniques for analysing supermarket loyalty card data (project)**

The arrival in recent years of supermarket loyalty cards has prompted the need to analyse vast sales databases. But given the limited availability of personal customer data associated with each sale, can statistical analyses make profitable use of these databases? Various statistical modelling concepts will be explored. A partial database from an abandoned system is available to test various statistical techniques.

Reference: Scoring Points by Clive Humby & Terry Hunt (Koran Page 2003)

Anybody interested in this proposed project should contact me as soon as possible in order for me to assess the computing aspects associated with the large volume of data involved (it would be advisable not to wait until Options Day).

## **Compilation of web pages to calculate sample size requirements for studies with standard and non-standard outcome data (project)**

When completing grant applications to any major funding body, researchers across the University are now required to produce and justify the sample size(s) that they intend to use.

Based on a specific hypothesis (typically involving treatment A vs. treatment B) within a proposed study, and given a specific size (usually 5%) and power (sometimes 90%, often 80%, occasionally lower), the corresponding required sample size is calculated according to the nature of the data which are usually continuous (e.g. test score out of 100, white cell count) or categorical (e.g. adverse event: yes/no). However when the primary outcome is ordinal (e.g. how do you feel: great/OK/sick as a parrot) different inputs and calculations are required.

A web based application has been created for a number of scenarios (including the complex latter one): [<http://www.maths.surrey.ac.uk/UserFiles/image/maths/bin/SSA.html>]

A sound knowledge of statistical theory is needed; the project would involve:

1. Expanding the scenarios in the application to address correlation hypotheses.
2. Producing a project report (effectively a manual) explaining what the web pages do and the statistical calculations involved.

Requirements :

1. Good knowledge of basic statistical tests, including non-parametric tests (e.g. the Mann-Whitney U test)
2. Some experience of web-based programming languages
3. Clear and concise English writing ability

Benefits: Would look good on CV if applying for any job in the field of research, but especially in medical statistics or statistical computing.

## **An epidemiological study of rheumatoid arthritis encompassing regional demographic risk factors (project)**

Rheumatoid arthritis is said to have a prevalence of 1% in the United Kingdom, mostly affecting elderly people. Yet clinicians have little idea as to what triggers the condition, despite knowing more and more about the associated immunology (t-cell functions, etc.). Looking at regional variations may help unravel the mystery.

The aim of this project is to investigate potential risk factors for rheumatoid arthritis using demographic data for different areas of the United Kingdom. For instance, if excessive manual work

leads to rheumatoid arthritis, then areas with high numbers of people employed in industrial work would be expected to display higher prevalences of rheumatoid arthritis. Alternatively, if lack of exercise leads to rheumatoid arthritis, then areas with high numbers of white collar workers might be expected to display higher prevalences. Other demographic factors will also be considered.

Simple epidemiological techniques such as age-specific morbidity rates [see Chapter 15 of Essentials of Medical Statistics by Betty Kirkwood (Blackwell 2001)] will be used to summarise the data, followed by regression techniques to model potential risk factors.

Requirements :

1. Willingness to learn the basic concepts of epidemiology
2. Willingness to perform a brief literature search on the relevant topic
3. Ability to learn a statistical computing package e.g. SAS or SPSS

Benefits : Would look impressive on CV if applying to do an M.Sc. in Medical/Social Statistics, or if applying for a statistics post within a medical research group or within a health department.

*N.B. Project availability uncertain owing to data acquisition problems.*

Please email P.Williams@surrey.ac.uk for an update.

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**Supervisor: Claudia Wulff**

**Stochastic Processes - Basic Limit and Perron Frobenius theorem**  
(Literature review spring semester only)

In the lecture course *Stochastic Processes MAT2003* the Basic Limit Theorem which states convergence to a unique stationary distribution for aperiodic, irreducible, positive recurrent Markov chain was stated, but not proved. In this literature review this theorem should be proved and a generalization of it, the Perron Frobenius theorem, should be studied.

Prerequisite: Stochastic Processes MAT2003

Literature: J.R. Norris, Markov chains, J. Hofbauer/ K. Sigmund "*The theory of evolution and dynamical systems*".

**Complex Analysis - Conformal Maps** (Literature review spring semester only)

A conformal map is a transformation that preserves angles. Analytic complex functions are conformal if they have nonvanishing derivatives. Such maps are used a lot in applications like fluid dynamics.

Prerequisite: Functions of a Complex Variable MAT2012

**ODEs - Basics of Chemical reaction networks** (Literature review spring semester only)

Chemical reaction graphs can be modelled by ordinary differential equations (ODEs) using mass action kinetics. Criteria for the existence of stable stationary solutions of the corresponding ODEs should be investigated in this literature review.

Prerequisite: Ordinary Differential Equations MAT2007

## ODEs - Numerical bifurcation analysis (Literature review spring semester only)

Numerical methods for the continuation of equilibria of parameter-dependent vectorfields will be studied, and techniques for the detection of computations should be reviewed in this project. Bifurcations mark a qualitative change of behaviour of a system, e.g., an equilibrium solution might disappear after a critical threshold in an external parameter is passed, or it might lose stability and instead stable periodic oscillations arise. It is advisable to choose the module *MAT3007 Chaos and Fractals* along with this review.

Prerequisites: Ordinary Differential Equations MAT2007, Numerical and Computational Methods MAT2001.

## ODEs - Lyapounov-Centre Theorem (Literature review spring semester only)

The Lyapounov-Centre Theorem states that near an equilibrium of a Hamiltonian system which has a pair of (non-resonant) purely imaginary eigenvalues there is a family of periodic orbits. In first approximation these are periodic orbits of the linearization at the equilibrium. In this review the Lyapounov-Centre Theorem should be proved and some examples should be studied. It is advisable to take the module MAT3008 "Lagrangian and Hamiltonian Dynamics" and "MAT3007 Chaos and Fractals" along with this review along with this review.

Prerequisites: Ordinary Differential Equations MAT2007

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## Supervisor: Karen Young

### Analysis of data from bio-medical experiments (project/literature review)

Bio-medical experiments often generate lots of data that are hard to analyse without the use of statistics. David Lovell from PGMS has suggested some possible ideas for projects, which can be undertaken under supervision of one of the statisticians above. Any such project will involve extending the techniques learnt in the second year statistics courses in order to analyse a set of real data. If you are interested, contact Karen Young or Janet Godolphin in first instance.

### A review of methods for circular data (literature review)

This literature review would look at different approaches to analysing circular data as used in departure direction of birds from a given release points, wind and ocean current directions.

### Understanding the error in predicting theft claims from ignoring the spatial correlation between claims in both time and space (location) (Project)

Assuming that all theft claims within a specified region ( a collection of contiguous full/sector/outward postal codes) are independent (i.e. correlation = 0) may be causing significant error in predicting theft claims experience. Quantifying the impact of any spatial independence may lead us to change our how we update our models with recent claims data which are both close in both time and space (location).

The Project will investigate a range of techniques for estimating the spatial dependence of theft claims in a specified region. e.g. investigating different forms of variograms, spatial distance measures and spatial correlation statistics.

This project would complement a final year module in spatial statistics, or could be carried out over the whole year beginning with a study of spatial data analysis principles/techniques for either

”spatial point processes” or ”spatial areal units”.

**Understanding how propensity models for conversion and retention degrade over time with a view to determining the optimal recalibration frequency. (Project)**

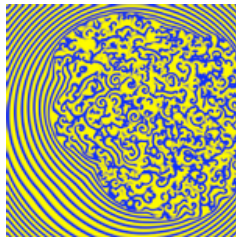
We currently use propensity models to predict the conversion of a quote and to predict retention of existing policies throughout the policy year and at renewal date. The accuracy of these models reduce over time, but how long is it before a significant drop off in predictive power is observed.

The Project will investigate a range of statistical measures for estimating model degradation which will determine an optimal frequency for recalibrating a specific propensity model.

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**Supervisor: Sergey Zelik**

**Partial differential equations - Spatio-temporal Chaos (project/literature review)**



Chemical reactions frequently exhibit chaotic/random fluctuations in both space and time, known as *spatio-temporal chaos*. This project/literature review will look at physical experiments and mathematical models where spatio-temporal chaos occurs. This project/literature review would be excellent for any student taking the *Chaos & Fractals* course.

**Functional Analysis - Applications (project/literature review)**

This project/literature review will be excellent for any student thinking of taking *Introduction to Function Spaces* in the third year. You will look at applications of functional analysis to the solutions of ordinary differential equations and partial differential equations.

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**Supervisor: Aleksey Andreev (Physics)**

**Effective solution of the eigenvalue problem for a sparse matrix (project)**

The aim of the project is to investigate available algorithms for effective numerical solution of the eigenvalue problem for a sparse matrix arising while solving Schrödinger's equations in three dimensions. The applications include modelling of basic elements (quantum bits - qubits) for quantum computers and potentially may involve some interaction with industry (Hitachi Cambridge Laboratory). Currently several PhD students are working of the physics side of the project. The student will be provided with a sparse matrix generator and the project work will involve: 1) a short literature review of available methods for sparse eigenvalue problem solution, with the aim of understanding the available algorithms, 2) investigation of the effectiveness of algorithms using available packages, 3) to study the possibility of effective parallel computing using these algorithms. The student will be working with FORTRAN codes, mainly using available packages, so some (very basic) initial knowl- edge of FORTRAN or another language would be helpful. The project will involve work on a powerful

computer cluster and possibly on a vector supercomputer available in the ATI.

### **Parallel computation of the eigenvalues for large matrices (project)**

The project is to look at the effective implementation of parallel computation of the eigenvalues and vectors of a large “dense” matrix arising from the solution of the Schrödinger’s equation using Fourier transform techniques. The applications of the work include modelling of single photon sources based on artificial atoms for quantum cryptography and may involve some interaction with industry (Qinetiq, Hitachi). Currently, several PhD students are working on the physics side of the project. The student will be provided with a generator of large dense matrices and the project will involve work with available packages (i.e. LAPACK family) for effective parallel computation of the eigenvalues and eigenvectors. The work would develop numerical computation skills (FORTRAN) and provide unique experience of working on a modern supercomputer.

### **Simulation of quantum computer operation with random fluctuations of a qubit (project)**

Type of project: short review and writing a computer program, analysis of the results. An operation of the qubits (quantum analogy of the bit), which is the basic element of the quantum computer, can be represented from a mathematical point of view as a linear transformation in linear space. In reality a random fluctuation is possible, which with a certain probability would slightly change the results of the linear transformation. During multiple linear transformations these random, small errors would accumulate. The purpose of the project is to write a computer program that would simulate this situation and to study the dependence of the qubits’ performance on the size and probability of these random fluctuations.

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